# Motor Equations

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## 1 Motor Equations

Motor Constants:

- $R_i$ - Motor resistance (ohms)
- $I_o$ - No-load Current (amps)
- $K_v$ - RPM/Volt
- $K_t$ - Oz-in/amp

Propeller Constant:

- $K_p$ - RPM/(Oz-in)

Electrical variables

- $V_{in}$ - Input voltage (volts)
- $I_{in}$ - Input current (amps)
- $V_m$ - Virtual voltage across motor (volts)
- $I_m$ - Virtual current through motor (amps)
- $P_{in}$ - Input power (watts)
- $P_{out}$ - Output power (watts)

Mechanical variables

- $f$ - Rotation rate (rotations/min)
- $\tilde{f}$ - Rotation rate (rotations/sec)
- $\tau$ - Torque (oz-in)
- $\tilde{\tau}$ - Torque (Newton-meters)

$$
\begin{aligned}
P_{in} &= V_{in} I_{in} \\
V_m &= V_{in} - I_{in} R_i \\
I_m &= I_{in} - I_o \\
P_{out} &= V_m I_m
\end{aligned}
$$

Conversion from Newton-meters to foot-pounds.

$$
\tau = \tilde{\tau} \frac{0.737562 \text{ ft-lb}}{\text{N-m}} \frac{16 \text{ oz}}{\text{lb}} \frac{12 \text{ in}}{\text{ft}}
$$

$$
\begin{aligned}
f &= K_v V_m \\
P_{out} &= 2\pi \tilde{f} \tilde{\tau} \\
P_{out} &= 2\pi \left( \frac{K_v V_m}{60} \right) \left( \frac{\tau}{0.737562 * 12 * 16} \right)
\end{aligned}
$$

From here we have that

$$
\begin{aligned}
V_m (I_{in} - I_o) &= 2\pi \left( \frac{K_v V_m}{60} \right) \left( \frac{\tau}{0.737562 * 12 * 16} \right) \\
I_m &= 2\pi \left( \frac{K_v}{60} \right) \left( \frac{\tau}{0.737562 * 12 * 16} \right) \\
K_t &= \frac{\tau}{I_m} \\
&= \left( \frac{60 * 0.737562 * 12 * 16}{2\pi K_v} \right)
\end{aligned}
$$

So the propeller constant in units of RPM/(oz-in) is given by

$$
\begin{aligned}
K_p &= \frac{V_m K_v}{I_m K_t} \\
&= \frac{V_m}{I_m} \frac{2\pi K_v^2}{60 * 0.737562 * 12 * 16}
\end{aligned}
$$