PURDUE

ECE 63700

Exam #1, February 20, Spring 2026

NAME _____

PUID _____

Exam instructions:

- A fact sheet is included at the end of this exam for your use.
- You have 50 minutes to work the exam.
- This is a closed-book and closed-note exam. You may not use or have access to your book, notes, any supplementary reference, a calculator, or any communication device including a cell-phone or computer.
- You may not communicate with any person other than the official proctor during the exam.

To ensure Gradescope can read your exam:

- Write your full name and PUID above and on the top of every page.
- Answer all questions in the area designated for each problem.
- Write only on the front of the exam pages.
- DO NOT run over to the next question.

Name/PUID: **Key**

Problem 1.(30pt) Starter Problems

Answer each problem below for $a, b \in \Re$ with $a > 0$ and $b > 0$.

Problem 1a) Give the CTFT of $s(t) = \text{rect}(at)$.

Solution:

$$S(f) = \frac{1}{|a|}\text{sinc}(f/a)$$

Problem 1b) Give the CTFT of $s(t) = \text{sinc}(at)$.

Solution:

$$S(f) = \frac{1}{|a|}\text{rect}(f/a)$$

Problem 1c) Give the CSFT of $s(x, y) = \delta(x)$.

Solution:

$$S(u, v) = \delta(v)$$

Problem 1d) Give the CSFT of $s(x, y) = \text{sinc}(ax)\,\text{sinc}(by)$.

Solution:

$$S(u, v) = \frac{1}{ab}\text{rect}(u/a)\text{rect}(vb)$$

Problem 1e) Calculate $s(t) = \text{sinc}(at) * \text{sinc}(bt)$.

Solution:

$$s(t) = \frac{1}{\max(a, b)}\text{sinc}\left(\min(a, b)t\right)$$

Name/PUID: **Key**
Problem 2.(30pt) DSFT and DTFT

Let $g(m,n)$ be a 2D discrete space function with a DSFT defined by

$$G(\mu,\nu) = \text{DSFT}\left[g(\cdot,\cdot)\right] = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} g(m,n)e^{-j\{\mu m+\nu n\}}$$

$$R(m,\nu) = \text{DTFT}_r\left[g(\cdot,\cdot)\right] = \sum_{n=-\infty}^{\infty} g(m,n)e^{-j\nu n}$$

$$C(\mu,n) = \text{DTFT}_c\left[g(\cdot,\cdot)\right] = \sum_{m=-\infty}^{\infty} g(m,n)e^{-j\mu m}$$

$$p(m) = \sum_{n=-\infty}^{\infty} g(m,n)$$

$$P(\omega) = \sum_{m=-\infty}^{\infty} p(m)e^{-j\omega m}$$

Problem 2a) Calculate a simplified expression for $R(m,0)$. What does it mean?

Solution:

$$R(m,0) = \sum_{n=-\infty}^{\infty} g(m,n)$$

The zero frequency (i.e., the DC frequency) for the m^{th} row corresponds to the sum over the m^{th} row.

Problem 2b) Calculate a simplified expression for $C(0,n)$. What does it mean?

Solution: The zero frequency (i.e., the DC frequency) for the n^{th} column corresponds to the sum over the n^{th} column.

Problem 2c) Prove that $\text{DSFT}\left[g(\cdot,\cdot)\right] = \text{DTFT}_c\left[\text{DTFT}_r\left[g(\cdot,\cdot)\right]\right]$.

Solution:

$$\begin{aligned}
\mathrm{DTFT}_c\left[\mathrm{DTFT}_r\left[g(\cdot,\cdot)\right]\right] &= \mathrm{DTFT}_c\left[\sum_{n=-\infty}^{\infty} g(m,n)e^{-j\nu n}\right] \\
&= \sum_{m=-\infty}^{\infty}\left[\sum_{n=-\infty}^{\infty} g(m,n)e^{-j\nu n}\right]e^{-j\mu m} \\
&= \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} g(m,n)e^{-j\nu n}e^{-j\mu m} \\
&= \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} g(m,n)e^{-j\{\nu n+\mu m\}} \\
&= G(\nu,\mu)
\end{aligned}$$

Problem 2d) Prove that $p(m) = R(m,0)$.

Solution:

$$\begin{aligned}
R(m,0) &= \sum_{n=-\infty}^{\infty} g(m,n)e^{-j0n} \\
&= \sum_{n=-\infty}^{\infty} g(m,n) \\
&= p(m)
\end{aligned}$$

Problem 2e) Prove that $P(\mu) = G(\mu,0)$.

Solution:

$$\begin{aligned}
P(\mu) &= \sum_{m=-\infty}^{\infty} p(m)e^{-j\mu m} \\
&= \sum_{m=-\infty}^{\infty} R(m,0)e^{-j\mu m} \\
&= \sum_{m=-\infty}^{\infty}\left\{\sum_{n=-\infty}^{\infty} g(m,n)e^{-j0n}\right\}e^{-j\mu m} \\
&= \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} g(m,n)e^{-j(0n+\mu m)} \\
&= G(\mu,0)
\end{aligned}$$

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Problem 3.(24pt) 2D Plane Waves

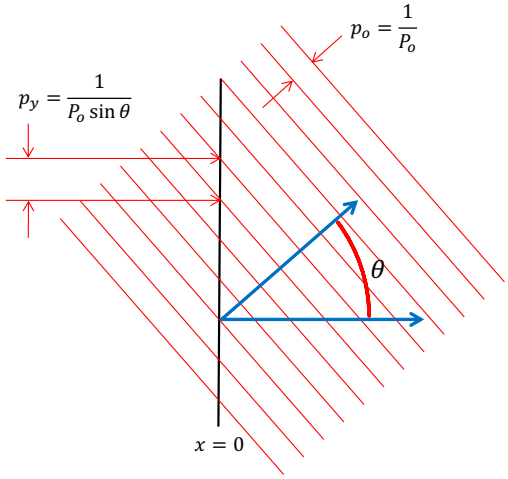
Consider a 2D plane wave with the form

$$g(x,y) = \cos\left(2\pi x P_o \cos\theta + 2\pi y P_o \sin\theta\right) \ .$$

The 2D wave is then sensed along the plane $x = 0$ to form the 1D signal $s(y) = g(0,y)$, and define $S(f) = CTFT\left[s(\cdot)\right]$.

Problem 3a) Sketch the 2D wave $g(x,y)$. Label the period of the wave $p_o = \frac{1}{P_o}$ and its angle θ on the sketched plot.

Solution:



Problem 3b) Calculate the function $s(y)$.

Solution:

$$s(y) = \cos(2\pi y v_o)$$

where $v_o = P_o \sin\theta$.

Problem 3c) Calculate the function $M(f) = |S(f)|$.

Solution: The function $s(y) = \cos(2\pi y v_o)$ where $v_o = P_o \sin\theta$. So we have that

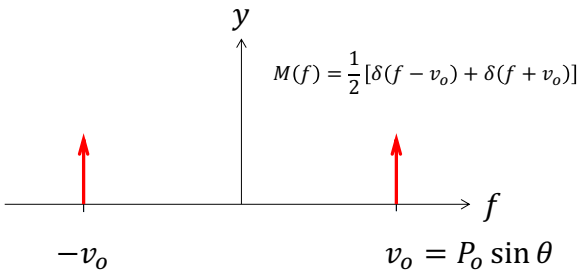
$$S(f) = \frac{1}{2}\left[\delta(f - v_o) + \delta(f + v_o)\right]$$

and

$$M(f) = |S(f)| = \frac{1}{2}\left[\delta(f - v_o) + \delta(f + v_o)\right]$$

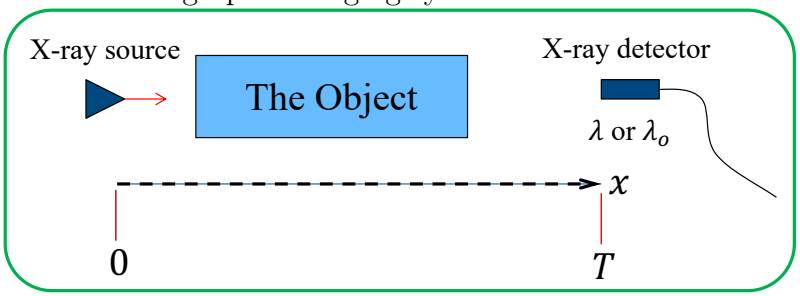
Problem 3d) Sketch the function $M(f)$ and label important parameters of the function.

Solution:



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Problem 4.(18pt) Tomography

Consider a transmission tomographic imaging system as shown below.



Then define:

- λ - The number of photons counted over a fixed time period when the object is present.
- λ_o - The number of photons counted over the same fixed time period when the object is removed.
- $\mu(x)$ - The density of the material in units of cm^{-1} at position $x \in [0, T]$.

Also define, $\bar{\lambda} = E[\lambda]$ and $\bar{\lambda}_o = E[\lambda_o]$ to be the expected measurements.

Problem 4a) Write an expression for the $\bar{\lambda}$ in terms of $\bar{\lambda}_o$ and $\mu(t)$.

Solution:

$$\bar{\lambda} = \bar{\lambda}_o \exp\left\{-\int_0^T \mu(t)dt\right\}$$

Problem 4b) Calculate an expression $y = f(\lambda, \lambda_o)$ so that $y \approx \int_0^T \mu(x)dx$.

Solution:

$$\lambda \approx \bar{\lambda} = \bar{\lambda}_0 \exp\left\{-\int_0^T \mu(t)dt\right\}$$
$$\approx \lambda_o \exp\left\{-\int_0^T \mu(t)dt\right\}$$

So we have that

$$\frac{\lambda}{\lambda_o} = \exp\left\{-\int_0^T \mu(t)dt\right\} \qquad = \lambda$$
$$-\log\left\{\frac{\lambda}{\lambda_o}\right\} = \int_0^T \mu(t)dt$$

From this, we have that

$$y = -\log\left\{\frac{\lambda}{\lambda_0}\right\} \approx \int_0^T \mu(t)dt$$

Problem 4c) Based on this, how should measurements from transmission CT scanners be preprocessed?

Solution: We first make two measurements, the object scan λ, and the blank scan λ_o. The blank scan is made with the object removed, and the object scan is made with the object in place.

Then the ration of the object an blank scan is computed to produce a transmission scan given by

$$T = \frac{\lambda}{\lambda_o}$$

The transmission scan falls in the range of $[0, 1]$.

The sinogram is then computed as the negative log of the transmission scan.

$$y = -\log T = -\log\frac{\lambda}{\lambda_o}$$

Fact Sheet

- Function definitions

$$\text{rect}(t) \triangleq \begin{cases} 1 & \text{for } |t| < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Lambda(t) \triangleq \begin{cases} 1-|t| & \text{for } |t| < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{sinc}(t) \triangleq \frac{\sin(\pi t)}{\pi t}$$

- CTFT

$$\begin{aligned} X(f) &= \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \\ x(t) &= \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df \end{aligned}$$

- CTFT Properties

$$x(-t) \overset{CTFT}{\Leftrightarrow} X^*(-f)$$

$$x(t-t_0) \overset{CTFT}{\Leftrightarrow} X(f) e^{-j2\pi ft_0}$$

$$x(at) \overset{CTFT}{\Leftrightarrow} \frac{1}{|a|} X(f/a)$$

$$X(t) \overset{CTFT}{\Leftrightarrow} x(-f)$$

$$x(t) e^{j2\pi f_0 t} \overset{CTFT}{\Leftrightarrow} X(f-f_0)$$

$$x(t)y(t) \overset{CTFT}{\Leftrightarrow} X(f) * Y(f)$$

$$x(t) * y(t) \overset{CTFT}{\Leftrightarrow} X(f)Y(f)$$

$$\int_{-\infty}^{\infty} x(t) y^*(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(f) Y^*(f) df$$

- CTFT pairs

$$\text{sinc}(t) \overset{CTFT}{\Leftrightarrow} \text{rect}(f)$$

$$\text{rect}(t) \overset{CTFT}{\Leftrightarrow} \text{sinc}(f)$$

For $a > 0$

$$\frac{1}{(n-1)!} t^{n-1} e^{-at} u(t) \overset{CTFT}{\Leftrightarrow} \frac{1}{(j2\pi f + a)^n}$$

- CSFT

$$\begin{aligned} F(u,v) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dx dy \\ f(x,y) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v) e^{j2\pi(ux+vy)} du dv \end{aligned}$$

- DTFT

$$\begin{aligned} X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \\ x(n) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega \end{aligned}$$

- DTFT pairs

$$a^n u(n) \overset{DTFT}{\Leftrightarrow} \frac{1}{1-ae^{-j\omega}}$$

$$(n+1)a^n u(n) \overset{DTFT}{\Leftrightarrow} \frac{1}{(1-ae^{-j\omega})^2}$$

- Rep and Comb relations

$$\text{rep}_T[x(t)] = \sum_{k=-\infty}^{\infty} x(t-kT)$$

$$\text{comb}_T[x(t)] = x(t) \sum_{k=-\infty}^{\infty} \delta(t-kT)$$

$$\text{comb}_T[x(t)] \overset{CTFT}{\Leftrightarrow} \frac{1}{T} \text{rep}_{\frac{1}{T}}[X(f)]$$

$$\text{rep}_T[x(t)] \overset{CTFT}{\Leftrightarrow} \frac{1}{T} \text{comb}_{\frac{1}{T}}[X(f)]$$

- Sampling and Reconstruction

$$\begin{aligned} y(n) &= x(nT) \\ Y(e^{j\omega}) &= \frac{1}{T} \sum_{k=-\infty}^{\infty} X\left(\frac{\omega - 2\pi k}{2\pi T}\right) \\ s(t) &= \sum_{k=-\infty}^{\infty} y(k) \delta(t-kT) \\ S(f) &= Y\left(e^{j2\pi fT}\right) \end{aligned}$$