EE 438 Midterm Exam #3

November 29, Fall 2007

Name: _____

Instructions:

- Follow all instructions carefully!
- This is a 50 minute exam containing **three** problems totaling 100 points.
- You **may not** use a calculator.
- You **may not** use any notes, books or other references.
- You may only keep pencils, pens and erasers at your desk during the exam.

Good Luck.

Fact Sheet

- Function definitions

$$\text{rect}(t) \triangleq \begin{cases} 1 & \text{for } |t| < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Lambda(t) \triangleq \begin{cases} 1-|t| & \text{for } |t| < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{sinc}(t) \triangleq \frac{\sin(\pi t)}{\pi t}$$

- Function types
  - *Right-sided:* $\exists N s.t. \forall n < N, x(n) = 0$
  - *Left-sided:* $\exists N s.t. \forall n > N, x(n) = 0$
  - *Causal:* $\forall n < 0, x(n) = 0$
  - *Anticausal:* $\forall n > 0, x(n) = 0$
  - *Two-sided:* {x(n) is not right-sided} and {x(n) is not left-sided}

- CTFS

$$\begin{aligned} a_k &= \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-j2\pi kt/T}dt \\ x(t) &= \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T} \end{aligned}$$

- CTFT

$$\begin{aligned} X(f) &= \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \\ x(t) &= \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df \end{aligned}$$

- CTFT Properties

$$x(-t) \overset{CTFT}{\Leftrightarrow} X(-f)$$

$$x(t-t_0) \overset{CTFT}{\Leftrightarrow} X(f)e^{-j2\pi t_0}$$

$$x(at) \overset{CTFT}{\Leftrightarrow} \frac{1}{|a|}X(f/a)$$

$$X(t) \overset{CTFT}{\Leftrightarrow} x(-f)$$

$$x(t)e^{j2\pi f_o t} \overset{CTFT}{\Leftrightarrow} X(f-f_o)$$

$$x(t)y(t) \overset{CTFT}{\Leftrightarrow} X(f) * Y(f)$$

$$x(t) * y(t) \overset{CTFT}{\Leftrightarrow} X(f)Y(f)$$

- CTFT pairs

$$\text{sinc}(t) \overset{CTFT}{\Leftrightarrow} \text{rect}(f)$$

$$\text{rect}(t) \overset{CTFT}{\Leftrightarrow} \text{sinc}(f)$$

$$e^{-at}u(t) \overset{CTFT}{\Leftrightarrow} \frac{1}{j2\pi f + a}$$

- DTFT

$$\begin{aligned} X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n} \\ x(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega \end{aligned}$$

- DTFT Properties

$$x(-n) \overset{DTFT}{\Leftrightarrow} X(e^{-j\omega})$$

$$x(n-n_o) \overset{DTFT}{\Leftrightarrow} X(e^{j\omega})e^{-j\omega n_0}$$

$$x(n)e^{j\omega_o t} \overset{DTFT}{\Leftrightarrow} X(e^{j(\omega-\omega_o)})$$

$$x(n)y(n) \overset{DTFT}{\Leftrightarrow} \frac{1}{2\pi}X(e^{j\omega}) * Y(e^{j\omega})$$

$$x(n) * y(n) \overset{DTFT}{\Leftrightarrow} X(e^{j\omega})Y(e^{j\omega})$$

- Z-transform

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} x(n)z^{-n} \\ x(n) &= \frac{1}{2\pi j}\oint_C X(z)z^{n-1}dz \end{aligned}$$

- DFT

$$\begin{aligned} X_k &= \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N} \\ x(n) &= \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j2\pi kn/N} \end{aligned}$$

- Sampling and Reconstruction

$$\begin{aligned} y(n) &= x(nT) \\ Y(e^{j\omega}) &= \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(\frac{\omega - 2\pi k}{2\pi T}\right) \\ s(t) &= \sum_{k=-\infty}^{\infty} y(k)\delta(t-kT) \\ S(f) &= Y\left(e^{j2\pi fT}\right) \end{aligned}$$

- Decimation and Interpolation

$$
\begin{aligned}
y(n) &= x(nD) \\
Y(e^{j\omega}) &= \frac{1}{D}\sum_{k=0}^{D-1} X\left(e^{j\frac{\omega-2\pi k}{D}}\right) \\
s(n) &= \sum_{k=-\infty}^{\infty} y(k)\delta(n-kL) \\
S(e^{j\omega}) &= Y(e^{j\omega L})
\end{aligned}
$$

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Problem 1.(34pt) Consider the following transfer functions for LTI systems with right-sided impulse responses,

$$\begin{aligned} G(z) &= (1 - az^{-1}) \\ H(z) &= \frac{1}{(1 - bz^{-1})} \end{aligned}$$

where a and b are two real numbers.

a) Specify the ROC for $G(z)$; calculate $g(n)$; and draw a flow diagram that implements this filter.

b) Specify the ROC for $H(z)$; calculate $h(n)$; and draw a flow diagram that implements this filter.

c) Specify the ROC for $F(z) = \frac{G(z)}{H(z)}$; calculate $f(n)$; and draw a flow diagram that implements this filter.

d) For what values of a and b is the system $F(z) = \frac{G(z)}{H(z)}$ stable? Justify your answer.

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Problem 2.(33pt)

Consider the stable LTI system $H(z) = \frac{1}{1-az^{-1}}$ with input $x(n)$ and output $y(n)$ where a is any real value.

a) For what values of a is the system right-sided? For what values of a is the system left-sided? Justify your answers.

b) Calculate $y(n)$ when $x(n) = e^{j\omega_o n}$.

c) Calculate $y(n)$ when $x(n) = \delta(n)$. (Hint: Your answer will depend on the value of a.)

d) Assuming that $a = 2$, specify a flow diagram that implements this system.

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Problem 3.(33pt)

a) Calculate $X_N(k)$, the N-point DFT of $x(n) = \cos\left(\frac{2\pi l}{N} n\right)$ for $0 \leq n < N$.

b) Sketch $X_N(k)$ for $0 \leq k < N$.

c) Calculate $X_N(k)$, the N-point DFT of

$$w(n) = \begin{cases} 1 & \text{if } 0 \leq n < L \\ 0 & \text{otherwise} \end{cases}$$

where $N \geq L$.

d) Sketch $|W_N(k)|$, for $L = 5$ and $N >> L$.
e) If $w(n)$ is being used to window a signal $x(n)$, what is the advantage of making N larger when L is fixed?

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