EE 438 Midterm Exam #2

October 25, Fall 2007

Name: _____

Instructions:

- Follow all instructions carefully!
- This is a 50 minute exam containing **three** problems totaling 100 points.
- You **may not** use a calculator.
- You **may not** use any notes, books or other references.
- You may only keep pencils, pens and erasers at your desk during the exam.

Good Luck.

Fact Sheet

- Function definitions

$$\text{rect}(t) \triangleq \begin{cases} 1 & \text{for } |t| < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Lambda(t) \triangleq \begin{cases} 1-|t| & \text{for } |t| < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{sinc}(t) \triangleq \frac{\sin(\pi t)}{\pi t}$$

- CTFS

$$\begin{aligned} a_k &= \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-j2\pi kt/T}dt \\ x(t) &= \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T} \end{aligned}$$

- CTFT

$$\begin{aligned} X(f) &= \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \\ x(t) &= \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df \end{aligned}$$

- CTFT Properties

$$x(-t) \overset{CTFT}{\Leftrightarrow} X(-f)$$

$$x(t-t_0) \overset{CTFT}{\Leftrightarrow} X(f)e^{-j2\pi t_0}$$

$$x(at) \overset{CTFT}{\Leftrightarrow} \frac{1}{|a|}X(f/a)$$

$$X(t) \overset{CTFT}{\Leftrightarrow} x(-f)$$

$$x(t)e^{j2\pi f_o t} \overset{CTFT}{\Leftrightarrow} X(f-f_o)$$

$$x(t)y(t) \overset{CTFT}{\Leftrightarrow} X(f) * Y(f)$$

$$x(t) * y(t) \overset{CTFT}{\Leftrightarrow} X(f)Y(f)$$

- CTFT pairs

$$\text{sinc}(t) \overset{CTFT}{\Leftrightarrow} \text{rect}(f)$$

$$\text{rect}(t) \overset{CTFT}{\Leftrightarrow} \text{sinc}(f)$$

$$e^{-at}u(t) \overset{CTFT}{\Leftrightarrow} \frac{1}{j2\pi f + a}$$

- DTFT

$$\begin{aligned} X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n} \\ x(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega \end{aligned}$$

- DTFT Properties

$$x(-n) \overset{DTFT}{\Leftrightarrow} X(e^{-j\omega})$$

$$x(n-n_o) \overset{DTFT}{\Leftrightarrow} X(e^{j\omega})e^{-j\omega n_0}$$

$$x(n)e^{j\omega_o t} \overset{DTFT}{\Leftrightarrow} X(e^{j(\omega-\omega_o)})$$

$$x(n)y(n) \overset{DTFT}{\Leftrightarrow} \frac{1}{2\pi}X(e^{j\omega}) * Y(e^{j\omega})$$

$$x(n) * y(n) \overset{DTFT}{\Leftrightarrow} X(e^{j\omega})Y(e^{j\omega})$$

- DTFT pairs

$$a^n u(n) \overset{DTFT}{\Leftrightarrow} \frac{1}{1-ae^{-j\omega}}$$

- DFT

$$\begin{aligned} X_k &= \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N} \\ x(n) &= \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N} \end{aligned}$$

- Sampling and Reconstruction

$$\begin{aligned} y(n) &= x(nT) \\ Y(e^{j\omega}) &= \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(\frac{\omega-2\pi k}{2\pi T}\right) \\ s(t) &= \sum_{k=-\infty}^{\infty} y(k)\delta(t-kT) \\ S(f) &= Y\left(e^{j2\pi fT}\right) \end{aligned}$$

- Decimation and Interpolation

$$\begin{aligned} y(n) &= x(nD) \\ Y(e^{j\omega}) &= \frac{1}{D}\sum_{k=0}^{D-1} X\left(e^{j\frac{\omega-2\pi k}{D}}\right) \\ s(n) &= \sum_{k=-\infty}^{\infty} y(k)\delta(n-kL) \\ S(e^{j\omega}) &= Y(e^{j\omega L}) \end{aligned}$$

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Problem 1.(34pt) Consider a discrete-time LTI system with input $x(n)$, output $y(n)$ and impulse response

$$h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-(\omega^2 + j\omega)} e^{j\omega n} d\omega \ .$$

a) Calculate the DTFT of the function $h(n)$, i.e. $H(e^{j\omega})$ for $|\omega| < \pi$.

b) Is the output of the system real-valued when the input is real-valued? Justify your answer.

c) Compute the $|y(n)|$, when the input is given by $x(n) = e^{j\omega_o n}$.

d) Compute the $y(n)$, when the input is given by $x(n) = \cos(\omega_o n)$.

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Problem 2.(33pt)



Consider the sampling system shown above.

a) Calculate expressions for the signals $Y(e^{j\omega})$, $Z(f)$, and $S(f)$ in terms of $X(f)$ and $H(f)$.

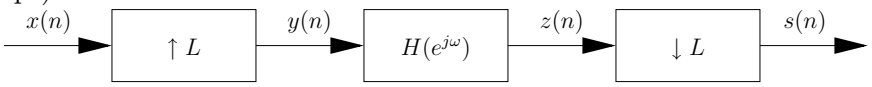
For parts b) and c) below, assume that $x(t)$ is band limited to $\frac{1}{2T}$.

b) Calculate an expression for $S(f)$ when $H(f)$ is a low pass filter with a cut off frequency of $\frac{1}{2T}$ and a gain of 1 in the pass band.

c) Specify a frequency response $H(f)$ so that $x(t) = s(t)$, and sketch the function $H(f)$.

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Problem 3.(33pt)



Consider the discrete-time system shown above with a general filter $H(e^{j\omega})$.

a) Calculate expressions for the signals $Y(e^{j\omega})$, $Z(e^{j\omega})$, and $S(e^{j\omega})$ in terms of $X(e^{j\omega})$ and $H(e^{j\omega})$.

b) Calculate an expression for the function $\frac{S(e^{j\omega})}{X(e^{j\omega})}$.

For parts c) and d) assume that $H(e^{j\omega})$ is a perfect LPF with a cut-off of π/L, and a pass-band gain of L.

c) Calculate the impulse response $h(n)$.

d) Calculate an expression for the function $\frac{S(e^{j\omega})}{X(e^{j\omega})}$.

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