EE 438 Midterm Exam #1

September 28, Fall 2007

Name: _____

Instructions:

- Follow all instructions carefully!
- This is a 50 minute exam containing **three** problems totaling 100 points.
- You **may not** use a calculator.
- You **may not** use any notes, books or other references.
- You may only keep pencils, pens and erasers at your desk during the exam.

Good Luck.

Fact Sheet

- Function definitions

$$\text{rect}(t) \triangleq \begin{cases} 1 & \text{for } |t| < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Lambda(t) \triangleq \begin{cases} 1-|t| & \text{for } |t| < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{sinc}(t) \triangleq \frac{\sin(\pi t)}{\pi t}$$

- CTFS

$$\begin{aligned} a_k &= \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-j2\pi kt/T}dt \\ x(t) &= \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T} \end{aligned}$$

- CTFT

$$\begin{aligned} X(f) &= \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \\ x(t) &= \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df \end{aligned}$$

- CTFT Properties

$$x(-t) \overset{CTFT}{\Leftrightarrow} X(-f)$$

$$x(t-t_0) \overset{CTFT}{\Leftrightarrow} X(f)e^{-j2\pi t_0}$$

$$x(at) \overset{CTFT}{\Leftrightarrow} \frac{1}{|a|}X(f/a)$$

$$X(t) \overset{CTFT}{\Leftrightarrow} x(-f)$$

$$x(t)e^{j2\pi f_o t} \overset{CTFT}{\Leftrightarrow} X(f-f_o)$$

$$x(t)y(t) \overset{CTFT}{\Leftrightarrow} X(f) * Y(f)$$

$$x(t) * y(t) \overset{CTFT}{\Leftrightarrow} X(f)Y(f)$$

- CTFT pairs

$$\text{sinc}(t) \overset{CTFT}{\Leftrightarrow} \text{rect}(f)$$

$$\text{rect}(t) \overset{CTFT}{\Leftrightarrow} \text{sinc}(f)$$

$$e^{-at}u(t) \overset{CTFT}{\Leftrightarrow} \frac{1}{j2\pi f + a}$$

- DTFT

$$\begin{aligned} X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n} \\ x(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega \end{aligned}$$

- DTFT Properties

$$x(-n) \overset{DTFT}{\Leftrightarrow} X(e^{-j\omega})$$

$$x(n-n_o) \overset{DTFT}{\Leftrightarrow} X(e^{j\omega})e^{-j\omega n_0}$$

$$x(n)e^{j\omega_o t} \overset{DTFT}{\Leftrightarrow} X(e^{j(\omega-\omega_o)})$$

$$x(n)y(n) \overset{DTFT}{\Leftrightarrow} \frac{1}{2\pi}X(e^{j\omega}) * Y(e^{j\omega})$$

$$x(n) * y(n) \overset{DTFT}{\Leftrightarrow} X(e^{j\omega})Y(e^{j\omega})$$

- DTFT pairs

$$a^n u(n) \overset{DTFT}{\Leftrightarrow} \frac{1}{1-ae^{-j\omega}}$$

- DFT

$$\begin{aligned} X_k &= \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N} \\ x(n) &= \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N} \end{aligned}$$

- Sampling and Reconstruction

$$\begin{aligned} y(n) &= x(nT) \\ Y(e^{j\omega}) &= \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(\frac{\omega - 2\pi k}{2\pi T}\right) \\ s(t) &= \sum_{k=-\infty}^{\infty} y(k)\delta(t-kT) \\ S(f) &= Y\left(e^{j2\pi fT}\right) \end{aligned}$$

Name: _____

Problem 1.(34pt) Let $T[\cdot]$ be a Discrete-time Linear time invariant system with input $x(n)$ and output $y(n)$; so that $y(n) = T[x(n)]$.

a) Prove that $y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$ where $h(n) = T[\delta(n)]$.

b) Use the result of a) above to show that when $x(n) = e^{j\omega_o n}$, then $y(n) = Ce^{j\omega_o n}$, and calculate an expression for the complex number C in terms of $h(n)$. (Do not use the Fourier transform to do this.)

c) For this part, assume that $h(n)$ is real valued. Then calculate the output $y(n)$ when $x(n) = \cos(\omega_o n)$ in terms of the complex number $C = Ae^{j\phi}$ from part b) above. (Do not use the Fourier transform to do this.)

Name: _____

Problem 2.(33pt) Consider a continuous-time signal

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{1}{k^2+1} e^{j2\pi kt}$$

which is the input to a CT LTI system with transfer function $H(f)$ and output $y(t)$.

a) Calculate the CTFT of $x(t)$.

b) Is $x(t)$ a real valued or complex function? Is it an even function, odd function, or has no symmetry? Justify your answer.

c) Give an expression for $y(t)$ in terms of $H(f)$.

d) Give conditions which guarantee that the output of the system will be real valued.

Name: _____

Name: _____

Problem 3.(33pt) Consider the following recursive difference equation

$$y(n) = x(n) + ay(n-1)$$

with input $x(n)$ and output $y(n)$.

a) Show that the system represented by the difference equation is a linear system.

b) Show that the system represented by the difference equation is a time-invariant system.

c) Use the DTFT to calculate the frequency response $H(e^{j\omega})$.

d) Use the result of part c) to calculate the impulse response $h(n)$.

Name: _____